28th ICPIG, July 15-20, 2007, Prague, Czech Republic

Revisiting the Normal Cathode Fall Theory

V.P. Nagorny

Mattson Technology, 47131 Bayside Pkwy, Fremont, CA 94538, USA

The theory of the normal cathode fall is being reexamined. Free diffusion of high-energy electrons over wide area about $\lambda_{dif}(U/I)^{1/2} \gg d_{CF}$, where λ_{dif} is a diffusive length, U is the voltage across the cathode fall, I is the ionization potential and d_{CF} is the length of the cathode fall (CF), accompanied by efficient ionization rather than a drift through the CF region accompanied by an avalanche is suggested. As electrons have energy significantly larger than ionization potential, the ratio of ionizing to exciting collisions is very large σ_i/σ_{ex}, so moving back and forth between CF and negative glow regions electron can produce large number of ionizations in the CF. Clarified the importance of the negative glow region for the operation of the CF.

1. Introduction

The normal cathode fall (NCF) is usually treated in theory as an extension of the gas breakdown theory [1] for the non-uniform field. Indeed, in every book related to gas discharge one can find the same fluid based consideration where the particle balance in the cathode fall is provided by the secondary electron emission from the cathode (coefficient γ) following by electron avalanche, which produces $1/\gamma$ number of ions in this region. The only difference with common consideration of the gas breakdown is a nonuniformity of the electric field and that the size of the cathode fall (d_n) related to the discharge current, rather than the gap length L. One often uses expression[1]

$$\alpha = Ap\exp(-Bp/E) \qquad (1)$$

for the first Townsend coefficient, describing the electron avalanche spatial growth rate, where E is the local electric field, and p is the gas pressure. While this approach is adequate for consideration of the cathode fall region when its size d_{CF} is still large compared to the size of the NCF, $pd_{CF} \gg pd_n$, it is certainly not good for consideration of the NCF, where not only the size of the nonuniformity, but the whole region is only a few times larger than electron mean free path. Not surprisingly, that this approach leads to some inconsistencies and misconceptions. For example, one usually suggests that "the potential distribution would be ideal if the potential difference equal to the minimum breakdown voltage, $V_{\min}$, was concentrated over the corresponding length $pd_{\min}$ at the cathode", [1] this would ensure reproduction at minimum applied voltage (optimum E/p). This implies that any other distribution would require higher voltage to sustain. Indeed, application of this view to a nonuniform field of the NCF results in the voltage across NCF and its size exceeding $V_{\min}$ and $pd_{\min}$: $V_{NCF} = 1.1V_{\min} > V_{\min}$, $pd_{NCF} = 1.4\,pd_n > pd_n$. Table 1 shows experimentally measured in the discharges in air and argon with iron cathodes values of $V_{\min}$, V_n, $pd_{\min}$, pd_n and predicted values of V_{NCF} and pd_{NCF}, based on values of $V_{\min}$ and $pd_{\min}$. One can see that opposite to the prediction, values V_n, and pd_n are actually not larger, but rather significantly smaller than $V_{\min}$ and $pd_{\min}$.

Table 1 [1, 2, 10]

Gas/ Cath.	$V_{\min}$	$1.1V_{\min}$	V_n	$pd_{\min}$	$1.4\,pd_{\min}$	pd_n
air/*Fe*	330	363	269	0.57	0.8	0.52
Ar/*Fe*	265	291	165	1.5	2.1	0.33
O_2/*Fe*	410	451	290	0.5	0.7	0.31

This fact by itself, independently on specific values of coefficients A, B and γ tells that the theory misses some important elements.

Most of attempts to investigate the cathode fall kinetically - both analytically and numerically [3-7], were focused on the account of nonlocal character of the first Townsend coefficient α in a nonuniform field, and demonstrated capabilities of one or another numerical approach using helium discharge. They however did not investigate a large number of discrepancies between predictions based on that approach and observed characteristics of the NCF. For example, it is not clear how such short cathode fall can sustain itself (in many cases the ionization length λ_i is almost as large as d_n) – numerically found [5] α is about half of what is necessary for

[1] For noble gases one use $\alpha = Cp\exp(-D\sqrt{p/E})$.

the balance. Not clear why discharge in the same gas and different cathode materials results in very different average electric fields $V_n/(pd)_n$. Based on tables for V_n and pd_n [1,2] the ranges of $V_n/(pd)_n$ (in $V/cmTorr$) for limited combinations of gases and cathode materials are: $517-1608$ in air, $512-581$ in N_2, $935-1296$ in O_2, $500-517$ in argon, and $82-115$ in helium. Even more strange that pd_n and V_n depend differently and non-monotonically on the secondary emission coefficient (see Table 2).

Clearly, existing description of the NCF is incorrect *qualitatively* and has to be reconsidered. In this presentation we suggest a new idea, which may help explain some peculiarities of the NCF.

Table 2. V_n and pd_n ($cmTorr$) for different combinations of gases and cathode materials [1, 2].

Gas/ Cath.	air	N_2	O_2	He
Al	229/0.25	180/0.31	311/0.24	140/1.32
Fe	269/0.52	215/0.42	290/0.31	150/1.30
Cu	370/0.23	-	-	-
Mg	-	188/0.35	310/0.25	125/1.45

It is sometimes suggested that ion flux from the NG region to the CF significantly affects the balance of the cathode fall and thus has to be included when considering the CF region. As we show in Appendix, this is rarely the case, so we will ignore that effect.

2. Cathode Fall – Qualitative Consideration

The most noticeable feature of the NCF and Abnormal CF is very large strength of the electric field near the cathode [8]. It is so high that on one mean free path electrons born close to the cathode gain energy significantly exceeding ionization threshold and in some gases (He, Ar, …) they continue to gain energy from the electric field up to the very end of the CF region where the field is already weak [4,7]. In this case electrons obviously do not reach any kind of equilibrium before they leave the CF and one wonders how they produce enough ion-electron pairs in the CF region to sustain the discharge. To answer this question and to investigate how CF size depends on the gas characteristics we consider a simple model similar to one developed in [9].

We assume that electron collision cross-sections $\sigma_{mt}, \sigma_{ex}, \sigma_i$ are constant above their thresholds (zero threshold for σ_{mt}), $\sigma_{mt} \gg \sigma_{ex}, \sigma_i$, that electron mean free path ($\lambda \sim \lambda_{mt} = 1/N\sigma_{mt}$) is only a few times smaller than the size of the CF d, that the energy electron gains near the cathode on one λ, $\varepsilon \sim eV\lambda/d$, is large compared to the ionization threshold I and that $eV \gg I$. Typically $eV\lambda/d$ exceeds I by a factor of 2. For simplicity we will also assume everywhere that after ionization collision, the new electron has zero kinetic energy and the incident one has the rest. Later we will discuss these assumptions.

Under these assumptions electron motion has diffusive rather than drift character – electrons freely move along electric field as well as against it. Although electron quickly leaves the CF region, it also quickly returns back in it and may produce ionization anywhere along its path. If this ionization happens in the NG region, then the product (ion-electron pair) stays there, and does not participate in the CF balance, but if ionization happens in the CF, then the ion is collected at the cathode, and electron may actively participate in the discharge. Depending on the position (initial energy), new electron can produce many new pairs and contribute significantly to the CF balance or leave CF without producing any new pairs. The role of electric field in this model is only to give electrons initial energy kick, to reflect slow electrons, and to collect ions.

Compared to ionization rate in relatively small uniform field E, where high energy EEDF tail is small ($\propto \exp[-(I-W_{ex})\sqrt{3\sigma_{mt}\sigma_{ex}}N/eE]$, where W_{ex} is excitation threshold), and even at optimum ionization conditions ($E/p = B$) it takes electron to pass much larger potential difference than I to produce one pair ($\bar{e}eB/AI \gg 1$, $\bar{e} = 2.718...$), in our case EEDF is wide and ionization is very effective: the ratio of ionization to excitation collisions is simply σ_i/σ_{ex}, which in many gases is larger than unity. For example, in O_2, this ratio is about 5 in a wide range of electron energies. If too much of these ionizations happened in the CF, then a large misbalance leading to growth of the current would result in shortening the CF region. If the CF is too short, then most of ionizations do not contribute to the CF balance, current decreases and CF becomes larger. The balance in the CF and its size is determined by some kind of a convolution between CF and diffusion region of high energy electrons. If voltage applied to the gap increases, then depending on the change of the diffusion profile in the whole CF-NG region this convolution may increase (resulting in CF shortening) or decrease (leading to CF enlargement).

This consideration shows that the negative glow is an essential part for the operation of the CF, and if one removes it (wall too close to the cathode [5]), then electrons leaving the CF region will not return back (absorbed by the wall), and the CF may not even be able to sustain itself at any voltage. This is why the minimum breakdown $pd_{\min}$ is always larger than pd_n. Of course, 1D consideration cannot distinguish between normal and abnormal current/conditions in the CF, but it gives interpretation of the balancing mechanism in the CF.

3. Size of the Cathode Fall

As usual, one can estimate the size of the cathode fall from the ion balance in the CF region:

$$\gamma \int_{CF} N_i(x)dx = 1, \qquad (2)$$

where $N_i(x)$ is the number of ions produced in the interval $(x, x+dx)$ of the CF as the result of a single electron emitted from the cathode. Since the problem is stationary it can be simplified further – one needs to know only spatial distribution of electron/ion production. The density of electrons having energy ε, $F(\varepsilon,x)$, can be found from equation

$$-D\frac{\partial^2 F(\varepsilon,x)}{\partial x^2} = -\frac{vF(\varepsilon,x)}{\lambda_{il}} + S_\varepsilon(x), \qquad (3)$$

where $D = v\lambda_{mt}/3$, $\lambda_{il} = 1/Nv(\sigma_{ex}+\sigma_i)$, $v = \sqrt{2(\varepsilon - e\varphi(x))/m}$, and $S_\varepsilon(x)$ is the source term. For $\varepsilon < U$ the source $S_\varepsilon(x)$ has two terms

$$S_\varepsilon(x) = \frac{v'F(\varepsilon',x)}{\lambda'_{il}} + \delta(\varepsilon - \xi^{-1}(x))\int_{\varepsilon+I}^{U}\frac{v'F(\varepsilon',x)}{\lambda_i}d\varepsilon', \qquad (4)$$

where prime means that the value is taken at the energy $\varepsilon' = \varepsilon + I$ and $\xi = \xi(\varepsilon)$ is the point x where electron kinetic energy is zero. Since equation (3) is linear with respect to $F(\varepsilon,x)$ we can look for solution like

$$F(\varepsilon,x) = F_1(\varepsilon,x) + F_2(\varepsilon,x), \qquad (5)$$

where $F_1(\varepsilon,x)$, and $F_2(\varepsilon,x)$ are the solutions of equation (3) with appropriate source term.

For F_1 ($S_{1\varepsilon} = Nv'\sigma'_{il}F(\varepsilon',x)$) one can use the boundary conditions:

$$\left.\frac{\partial F_1}{\partial x}\right|_{x=\xi(\varepsilon)} = 0, \quad F_1(\varepsilon, x\to\infty) = 0, \qquad (6)$$

where $\xi(\varepsilon)$ is the point where electron kinetic energy is zero. Solving (3)-(4) for $S_\varepsilon(x) = \delta(x - x_s)$, we find the Green function $G_\varepsilon(x,x_s)$:

$$G_\varepsilon(x,x_s) = \sqrt{\frac{3\lambda_{il}}{v^2\lambda_{mt}}}\begin{cases}\mathrm{ch}\mu(x-\xi)e^{-\mu(x_s-\xi)}, & x<x_s\\ \mathrm{ch}\mu(x_s-\xi)e^{-\mu(x-\xi)}, & x_s<x\end{cases} \qquad (7)$$

$$F_1(\varepsilon,x) = \int_{\xi(\varepsilon)}^{\infty} S_{1\varepsilon}(x_s)G_\varepsilon(x,x_s)dx_s. \qquad (8)$$

Here $\mu = 1/\lambda_{dif} = \sqrt{3/\lambda_{mt}\lambda_{il}}$.

For $F_2(\varepsilon,x)$ ($S_{2\varepsilon} = a\delta(\varepsilon - \varepsilon(x))$), and $\varepsilon(x) > I$ one can find the following solution for $x \ge \xi' \equiv \xi + I/eE(\xi)$ (we assume $eE > 2I/d_{CF}$)

$$F_2(\varepsilon,x) = \frac{a\delta(\varepsilon-\varepsilon(\xi))}{2eE(\xi)}\sqrt{\frac{3m\lambda_{il}}{2I\lambda_{mt}}}e^{-\mu(x-\xi')}, \qquad (9)$$

and for $F(\varepsilon = U, x)$ in the region $x \ge x_I \equiv I/eE(0)$, we can use the following solution

$$F(U,x) = \frac{m\delta(\varepsilon-U)}{4\pi eI}\sqrt{\frac{3\lambda_{il}}{\lambda_{mt}}}\gamma j_i(0)e^{-\mu(x-x_I)}. \qquad (10)$$

Note, that $x_I = \xi(\varepsilon = U - I)$, and similar equalities can be written for other energies: $\xi'(\varepsilon) = \xi(\varepsilon - I)$. Equations (4)-(10) describe EEDF resulting from the cascade of ionizations/excitations initiated by secondary electrons starting from the cathode.

The solution $F_1(\varepsilon,x)$ describes redistribution of electrons in space associated with the energy loss - diffusive growth of the EEDF footprint (λ_{dif}), and its shift from the cathode ($\xi(\varepsilon)$). The other part, $F_2(\varepsilon,x)$ gives contribution to EEDF due to the newly born electrons. The difference between ionization within and outside of the CF is seen from the second term in (4), describing the zero kinetic energy electrons appearing due to ionization. If ionization collision occurs outside of the CF region $x_s > d_{CF}$, then $\varepsilon = \xi^{-1}(x_s) = 0$, and these electron and ion stay in place without any further effects. But if it occurs inside the CF region $x_s < d_{CF}$, new electron with energy $\varepsilon = \xi^{-1}(x_s)$ can produce more ionizations, and ion moves toward the cathode, where it is collected. The smaller x_s, the larger energy of the second electron, the more ionizations it can make.

The size of the cathode fall d_{CF} appears in these equations only in the condition (2), and through the second part of the source term, where only $x_s < d_{CF}$ makes contribution to the balance. The current density should be determined afterwards as the one providing the necessary size d_{CF} of the CF.

4. Summary

The proposed model suggests that in a high field of normal and abnormal cathode fall, high energy electrons diffuse in a wide area, moving back and forth between CF and negative glow regions and efficiently ionizing the gas. The ratio of ionizing to exciting collisions can be as large as σ_i / σ_{ex}, which in some cases is significantly larger than unity (oxygen). The negative glow region, which serves as diffusively reflective wall is very important for the operation of the CF. If one removes it, placing an absorbing wall too close, it may actually extinguish the discharge.

It is worth to estimate the contribution of high energy electrons into the balance (2). In a regular model electron emitted from the cathode on its way to the NG produces about $N \sim \ln(1/\gamma)$ ionizing collisions (generations), which is about 5-7 for $\gamma \sim 10^{-2} - 10^{-3}$. In our case, this number is about

$$N \sim \frac{L_{CF}}{\lambda_{dif}\sqrt{U/I}} \frac{U}{I} \frac{\sigma_i}{\sigma_{ex}+\sigma_i} \sim \frac{L_{CF}}{\lambda_{dif}} \sqrt{\frac{U}{I}} \frac{\sigma_i}{\sigma_{ex}+\sigma_i},$$

where L_{CF} is the size of the cathode fall, and all cross-sections should be calculated at high energy. For oxygen, using $\sigma_{mt} \sim (7-8)\times 10^{-16} cm^2$ $\sigma_{ex} \sim 6\times 10^{-17} cm^2$, $\sigma_i \sim (3-5)\sigma_{ex}$, $I \sim 12eV$ and Table 2 we obtain $N \sim 30$. In nitrogen this number drops to about 20, which is still very large. With such a "boost" from high energy electrons (even though most of the ionizations occur in the end of the CF, where the electric field is relatively weak), one needs only a small amplification (avalanche-like) from electrons born in the end of the CF to reach the balance (2).

Author is grateful to M.N. Shneider and G.V. Stupakov for useful discussions.

5. Appendix

Let us estimate conditions when the ion balance in the NG is provided by their diffusion to the CF region and when due to recombination, assuming that there is a positive column present, so that ions can only diffuse toward the cathode and the diffusion coefficient is not ambipolar. In molecular gases the recombination term can be estimated as $\dot{N}_{rec} \sim \beta n_m^2 l$, where β, and n_m are the recombination coefficient and maximum ion density in the NG, and l is the length of the NG region. The diffusion ion flux toward the CF is $\dot{N}_{dif} \sim n_m D / l$. Recombination dominates the balance if $n_m > n_{cr} = D/l^2\beta$. At $n_m = n_{cr}$, the ion current density at CF-NG boundary is $j_i \sim e\dot{N}_{dif} \sim eD^2/\beta l^3$. If it is small compared to the normal current density j_n, then one can always neglect it considering the balance in the CF. Ion diffusion coefficient is $D \sim \lambda_{cx} v_T$, where $v_T = (T/M)^{1/2}$ and λ_{cx} are ion thermal velocity and ion charge exchange path, T is the gas temperature, and $l \sim a\lambda_{dif} \sim a\sqrt{\lambda_{il}\lambda_{mt}}$ is NG length, where λ_{il} and λ_{mt} are electron inelastic and momentum transfer paths, $a > 1$. In molecular gases $\beta \sim \beta_0 = 2\times 10^{-7}$ cm^3/s, so if we use for the estimate $\lambda_{dif} \sim 0.3/p$, then $j_i \sim eD^2/\beta l^3 \sim ev_T \varepsilon^2 (v_T/a^3\beta\lambda_{dif}) \sim$ $0.3p(\varepsilon^2/a^3)(M_{Air}/M)\ mA/cm^2$, where p is in Torr, $\varepsilon \equiv \lambda_{cx}/\lambda_{dif} \sim 0.3 - 0.01 << 1$ and M_{Air}/M is the mass ratio of the air to the gas molecule. One can clearly see that this value is small compared to j_n, which is usually about $(0.1-0.5)p^2\ mA/cm^2$. Hence, one can ignore the ion flux from the NG into the CF.

6. References.

[1] Yu.P. Raizer "Gas discharge physics", (Springer-Verlag, Berlin, 1991).

[2] S.C. Brown "Basic Data of Plasma Physics", MIT Press, 1967; Springer, 1994.

[3] Yu.P. Raizer, M.N. Shneider – *Teplofizika Vysokikh Temperatur,* **29**,1041 (1991) - Russian

[4] V.I. Kolobov, L.D. Tsendin, *Phys.Rev.* **A46**, 7837 (1992).

[5] J.-P. Boeuf, E. Marode, *J. Phys. D, Appl. Phys*. **15**, 2169 (1982)

[6] V.A. Shveigert, I.V. Shveigert, *Fiz. Plazmy* **14**, 347 (1988)

[7] V.P. Nagorny, *Physica Scripta*, **T53**, 79 (1994).

[8] N.N. Khristov, *Fiz. Plazmy* **6**, 436 (1980).

[9] V.Y. Khasilev, V.S. Mikhailevskiy, G.N. Tolmachev, *Fiz. Plazmy* **6**, 430 (1980).

[10] V.A. Lisovskiy, S.D. Yakovin, V.D. Yegorenkov, Workshop on FLTPD, March 25-29, 2001, Rolduc, Netherlands (available at www.phys.tue.nl/FLTPD/poster/lisovskiy.pdf)