Plasma Sources Sci. Technol. **6** (1997) 212–219. Printed in the UK PII: S0963-0252(97)82360-2

Effective secondary emission coefficient in a high-pressure noble gas

V P Nagorny and P J Drallos

Plasma Dynamics Corporation, 417 E 8 Mile Road, Hazel Park, MI 48030, USA

Received 29 July 1996, in final form 20 February 1997

Abstract. The back-scattering of secondary electrons toward the cathode in a high-density noble gas and the resulting effective secondary (electron) emission coefficient, ESEC, is evaluated using a kinetic approach. The behaviour of the ESEC in a mixture of noble gases is also discussed. The obtained results can be applied to different sources of secondary electrons—ions, photons, etc. We provide a comparison of obtained expressions with experiments.

1. Introduction

The phenomenon of secondary electron emission from a surface under the action of a primary particle (photon, ion, excited atom, etc) plays an important role in gas discharge physics. This role is reflected in the theory by the effective secondary emission coefficient (ESEC) γ: the number of electrons leaving the cathode surface per impinging primary particle [1, 2]. In a high-density gas the γ grows with E/p at low E/p values (where E is the electric field and p is a gas pressure), and reaches a constant value at high E/p [3–6]. The mechanism responsible for this dependence was identified as a reflection of a part of the secondary electrons from the gas atoms back to the cathode surface [1]. Based on J J Thomson's simple hydrodynamic model, Loeb proposed a qualitative expression describing such a behaviour [1, 2]: $\gamma = \gamma_i[4v_d/(\bar{v}_0 + 4v_d)]$, where γ_i is the vacuum value of the secondary emission coefficient, $\bar{v}_0$ is the average speed of electrons just emitted from the cathode and v_d is the electron drift velocity.

Using a kinetic approach for electrons we have considered analytically [7] the case when the voltage across the gap does not exceed the excitation threshold of an atom and found that electron scattering at large distances from the cathode ($L \gg \lambda_e$) contributes to returning electrons to the cathode and hence, to the inhibition of the ESEC, γ. In principle, the γ value for the discharge is determined by the electron transport in the whole gap, not only near the cathode. The value of γ is determined by the close vicinity of the cathode surface only if the electron transport cross-section σ_{tr} drops fast enough with the electron kinetic energy.

In this report we consider another case, when the voltage across the gap significantly exceeds the excitation/ionization potential. As we did before [7], we limit our consideration to a weakly ionized noble gas under conditions when the electron mean free path λ_e is small compared to the scale-length L of the problem, and electric field E is weak, so that $eE\lambda_e \ll W$, where W is the electron kinetic energy. We assume, though, that the electric field is high enough so that one can neglect the energy exchange between electrons and heavy particles in elastic collisions. For simplicity we consider only the case when the electric field is uniform and directed normally to the surface (compare with [7, 8]). Since the value of ESEC involves electron dynamics, it is natural to expect that its value will be different in the stationary and non-stationary cases. We will assume everywhere that the electric field in the whole region, which determines ESEC, is constant or changes much more slowly than the time of establishing a quasi-equilibrium state in the same region (later we will specify these conditions). In this sense we will consider stationary or slowly changing electric fields. Electron dynamics in a gas does not depend on the specific source of secondary electrons (ions, photons, etc), and the resulting expressions can be applied to any of these sources. However, to be specific we will consider only one such source—their liberation from the surface by the action of ions.

According to a common situation in noble gases we assume that the excitation and ionization thresholds are higher than the energy of secondary electrons near the surface [9]. Indeed, the cut-off of the energy spectrum of secondary electrons is about $W_i - 2A$, where W_i is the ionization potential of the gas atoms and A is a work-function of the surface (the maximum of the energy distribution function is at about half of this value [9]). Since the excitation threshold W_{ex} of all noble gases is only 4–5 eV lower than W_i, all or almost all of electrons have energy below W_{ex} when they leave the surface.

The paper is organized as follows. In section 2 we write the basic equations describing the problem. In section 3 we

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analyse the case of the noble gases and find the ESEC for them. Several analytic solutions of the kinetic equation, which are presented in this section, are of some value themselves as they can serve as benchmark solutions for testing numerical codes. In section 4 we analyse the ESEC for mixtures of noble gases, and, finally, in section 5 we give a short summary of the obtained results.

2. Basic equations

Let us consider a planar anode–cathode gap of width L and direct the z-axis normally to the cathode, so that the electric field is anti-parallel of the axis. Due to a symmetry of the problem we conclude that the gradient of the distribution function is also colinear with the z-axis. As was mentioned in the introduction we consider herein the case of a high-density gas or weak electric field, so that the electron mean free path is small compared to the scale-length of the problem, and the kinetic energy that electron gains between two collisions is small compared to its kinetic energy, $eE\lambda_e \ll W$. In this case the electron distribution function (EDF) is almost isotropic at distances from the electrodes exceeding a few λ_e and one can use for it a representation [10]

$$f(\boldsymbol{r}, \boldsymbol{v}, t) = f_0(\boldsymbol{r}, v, t) + \delta f \qquad (\delta f \ll f_0) \tag{1}$$

where isotropic $f_0(\boldsymbol{r}, v, t)$ and anisotropic δf parts satisfy the following equations [10]:

$$\frac{\partial f_0}{\partial t} = \frac{1}{3v^2}\left(v\frac{\partial}{\partial z} + \frac{eE}{m}\frac{\partial}{m}\right)\frac{v^2}{\nu_{tr}}\left(v\frac{\partial}{\partial z} + \frac{eE}{m}\frac{\partial}{\partial v}\right)f_0 + C_{il}(f_0) + C^1_{el}(f_0) \tag{2}$$

$$\delta f = -g\cos\theta \tag{3}$$

where $E = |E_z| = -E_z$

$$g(v) = \frac{1}{\nu_{tr}}\left(v\frac{\partial f_0}{\partial z} + \frac{eE}{m}\frac{\partial f_0}{\partial v}\right) \tag{4}$$

and θ is the angle between the velocity vector and the z-axis. Here $\nu_{tr} = N\sigma_{tr}v$ is the electron collision frequency, σ_{tr} is the electron–atom momentum transfer cross-section, $C_{il}(f_0)$ is an inelastic collision term, and $C^1_{el}(f_0)$ is the part of the electron elastic collision term describing the energy exchange between electrons and neutral gas atoms. We shall discuss the $C_{il}(f_0)$ and $C^1_{el}(f_0)$ later. The electron density n_e and electron current j_e are related to f_0 and δf, respectively:

$$n_e = 4\pi\int_0^\infty v^2 f_0\,\mathrm{d}v$$

$$j_e = -e\int v_z\,\delta f\,\mathrm{d}^3v = \frac{4\pi e}{3}\int_0^\infty v^3 g\,\mathrm{d}v. \tag{5}$$

It is convenient to choose new independent variables (z, ε) instead of (z, v), where

$$\varepsilon = mv^2/2 - eEz \tag{6}$$

is the electron energy, and we also choose $z = 0$ at the cathode (we remind the reader that our analysis is limited to a uniform electric field case). In the new variables (2) and (4) take the following form:

$$\frac{\partial f_0(z,\varepsilon)}{\partial t} = \frac{1}{3v}\frac{\partial}{\partial z}\frac{v^3}{\nu_{tr}}\frac{\partial}{\partial z}f_0(z,\varepsilon) + C_{il}(f_0) + C^1_{el}(f_0) \tag{7}$$

$$g(\varepsilon, z) = \frac{v}{\nu_{tr}}\frac{\partial f_0(z,\varepsilon)}{\partial z} \tag{8}$$

where v is considered as the function of (z, ε) according to (6). According to (6) $\varepsilon > 0$ at the cathode ($z = 0$), so that $\varepsilon > 0$ is a necessary condition for electron to return to the cathode, and for evaluating ESEC we need to know the electron distribution function *only in the energy range* $\varepsilon > 0$.

As we mentioned in the introduction, if the electric field is not very small, then one can neglect the energy exchange between electrons and neutral gas atoms ($C^1_{el}(f_0) = 0$). Indeed, an electron gains kinetic energy in the electric field with the rate $r_+ \sim eEv_{dr} \sim (eE)^2/(m\nu_{tr})$, where $v_{dr} \sim eE/(m\nu_{tr})$ is electron drift velocity, and loses it (due to elastic collisions) with the rate $r_- \sim (m/M)\nu_{tr}W$, where $W = mv^2/2$ is the electron kinetic energy, and M is a mass of a background gas atom. One can neglect the energy exchange in elastic collisions if $r_+ \gg r_-$. Comparing these rates we find that $r_+ \gg r_-$ in the whole range of electron kinetic energies up to excitation energy W_{ex}, when $eE\lambda_{tr} \gg \sqrt{m/M}W_{ex}$. Under this condition the electron reaches the excitation threshold without losing its energy. We assume here that the electric field satisfies this condition and neglect the $C^1_{el}(f_0)$ term in (7).

The inelastic collision term $C_{il}(f_0)$ consists of two parts: the scattering 'out' of an element ($\mathrm{d}\boldsymbol{r}$, $\mathrm{d}\boldsymbol{v}$) of a phase space ('sink' term) and scattering 'into' the element ($\mathrm{d}\boldsymbol{r}$, $\mathrm{d}\boldsymbol{v}$). As we mentioned in the introduction, the excitation threshold, W_{ex}, in noble gases is usually larger than the maximum of the energy spectrum of emitted electrons, W_{max}. Thus, after only one inelastic collision the electron leaves the initial energy range $0 < \varepsilon < W_{max}$ (energy becomes negative and the electron leaves the region within which it could return to the cathode), so that the scattering 'into' term is absent in this energy range, and for the inelastic collision term $C_{il}(f_0)$ in this range of energies we can write

$$C_{il}(f_0) = -\nu_{il}(v)f_0. \tag{9}$$

Here $\nu_{il}(v) = Nv(\sum_k \sigma_k(v) + \sigma_i(v))$ is the energy dependent frequency of inelastic collisions; $\sigma_k(v)$ is an excitation cross-section, $\sigma_i(v)$ is the ionization cross-section and N is the background gas density. It should be noted that $\varepsilon > 0$ is a necessary condition for electrons to reach the cathode, so that after one inelastic collision the electron leaves the initial energy range and automatically leaves the region within which it could still return to the cathode. The absence of the scattering -'in' term allows one to consider EDF and 'differential ESEC' (see later) independently for every energy layer (W_0, $W_0 + \mathrm{d}W_0$).

Since $W_{max} < W_{ex}$, electron kinetic energy at the surface, W_0, is not sufficient to excite an atom, and inelastic collision can occur only beyond certain distance from the cathode $z_I(W_0)$:

$$z_I(W_0) \equiv (W_{ex} - W_0)/eE \tag{10}$$

after the electron gains enough energy to excite an atom. For the distances from the cathode closer than $z_I(W_0)$, $C_{il}(f_0) = 0$. Beyond this point, $z > z_I(W_0)$, an electron with total energy W_0 can undergo inelastic collision. Using the energy conservation law (6) for the electron before it experiences an inelastic collision we can rewrite the collision term (9) in terms of energy and coordinate, $\nu_{il}(v) = \nu_{il}(W) = \nu_{il}(W_0 + eEz)$.

We shall discuss the boundary conditions for (7) in the next section.

3. Effective secondary emission coefficient for noble gases

As we consider here secondary electron emission from the cathode due to the action of ions striking the cathode, we identify the ESEC γ with the ratio of the electron and ion currents near the cathode surface. Since we consider every energy layer in the range $0 < \varepsilon < W_{max}$ independently, we will first find the 'differential' ESEC as a function of W_0 and electric field. Then, integrating the differential ESEC over the spectrum of emitted secondary electrons we shall find the total ESEC as a function of electric field. The width of the energy spectrum will serve as a parameter, characterizing the surface–gas interaction.

The stationary solution of (7) (in the energy range $\varepsilon > 0$) differs from zero only at the energy equal to the initial energy of electrons, $\varepsilon = W_0$. Accordingly, the general stationary solution for f_0 is

$$f_0(z, \varepsilon) = F(z)\delta(\varepsilon - W_0) = F(z)\delta(W - eEz - W_0) \quad (11)$$

where $F(z)$ satisfies the following equation:

$$\frac{1}{3v}\frac{\partial}{\partial z}\frac{v^3}{\nu_{tr}}\frac{\partial}{\partial z}F = \nu_{il}(v)F. \quad (12)$$

Substituting (11) into expressions (5), we find the electron density and current constituted by electrons of the energy W_0:

$$n_e = 2\pi F v \qquad j_e = \frac{2\pi e v^3}{3\nu_{tr}}\frac{\partial F}{\partial z} \quad (13)$$

or

$$j_e = \frac{e v^3}{3\nu_{tr}}\frac{\partial}{\partial z}\frac{n}{v} \quad (14)$$

v is considered here as a function of z given by (6) with $\varepsilon = W_0$.

Let us now formulate the boundary conditions for the $F(z)$. One of them is obvious:

$$F(z) \to 0 \qquad \text{when } z \to \infty. \quad (15)$$

This condition reflects the fact that sooner or later every electron which has large enough kinetic energy experiences an inelastic collision. Although the distance between electrodes is finite, we can also use this boundary condition if the voltage applied to the gap is much larger than the excitation threshold.

Let us now formulate the boundary condition at the cathode. The current to the cathode surface consists of two terms: the current of the emitted electrons $G(W_0)\gamma_i j_i$ and the current of electrons returning from the bulk of the discharge volume. Here we introduced the differential vacuum secondary emission coefficient $G(W_0)\gamma_i$, where the function $G(W_0)$ shows what part of the emitted electrons has the initial energy in the range $(W_0,\ W_0 + \mathrm{d}W_0)$, and satisfies the normalization condition

$$\int_0^{W_{max}} G(W_0)\,\mathrm{d}W_0 = 1. \quad (16)$$

There is a temptation to present the current of electrons returning back to the surface as constituted by the 'hemisphere' of the electron distribution f, near the cathode. In our approximation (1) this would give (remember that with our choice of the direction of the electric field the current is negative)

$$en_c v_0/4 \quad (17)$$

where n_c is the electron density in the vicinity of the cathode (at a distance of a few mean free paths λ_e), and $v_0 = \sqrt{2W_0/m}$ the electron speed near the cathode. This expression, however, does not take into account (i) scattering of these electrons back into the discharge gap at distances smaller than λ_e (i.e., a phenomenon that would reduce the flux with respect to the expression (17)) or (ii) back-scattering of the just emitted electrons (i.e., a phenomenon that would increase the flux to the cathode with respect to (17)). The relative role of these two phenomena depends on the peculiarities of the differential scattering cross-section and on the angular distribution of the emitted electrons. In other words, expression (17) should in fact be multiplied by some unknown numerical factor ζ of the order of unity (which can be found by the numerical solution of the Boltzmann equation at the distances $\sim \lambda_e$ from the wall).

There exists a special case when ζ is just equal to 1. This is the case of isotropic distribution of emitted electrons and weak electric field. Then the situation near the cathode is only slightly (in the parameter $eE\lambda_e/W_0$ and λ_e/L) different from the case of the isotropic electron distribution confined by the perfectly reflecting wall, i.e., the case when the distribution function is isotropic even at $z \ll \lambda_e$. Accordingly, for an isotropic distribution of secondary electrons and small electric fields, we have the following boundary condition at the cathode:

$$j_e(W_0) = G(W_0)\gamma_i j_i + en_c v_0/4. \quad (18)$$

Here j_e and n_c should be expressed in terms of the function $F(z)$ using (13)–(14). In (18) we used the notation $j_e(W_0)$ to remind the reader that this current is related to a specific energy W_0, while j_i is the total ion current density. Note that at small electric fields the two terms in the r.h.s. of (18) almost exactly compensate each other so that $j_e \ll G(W_0)\gamma_i j_i$.

We assume that the electric field (E/p) is not very strong, so that after an electron reaches the excitation threshold it does not gain much energy before it undergoes an inelastic collision. In other words we assume that $|W_{ex}\,\partial f_0(W)/\partial W|_{W_{ex}}| \ll f_0(W_{ex})$. In this case for the frequency of inelastic collisions above the threshold we can use the simple approximation

$$\nu_{il}(v) = \partial\nu_{il}/\partial W|_{W=W_{ex}}(W - W_{ex}) \\ \equiv \nu'_{il}(W_{ex})(W - W_{ex}) \qquad W > W_{ex} \tag{19}$$

where the prime means the energy derivative. Below the threshold, of course,

$$\nu_{il}(v) = 0 \qquad W < W_{ex}. \tag{20}$$

Substituting (6) and (10) into (19) we obtain for $\nu_{il}(v)$ and $z \geq z_I$

$$\nu_{il}(v) = \nu'_{il}(W_{ex})(W_0 + eEz - W_{ex}) \\ = \nu'_{il}(W_{ex})(z - z_I)eE(z_I). \tag{21}$$

Saving only the main term in the left-hand side of (12) (the one containing the second derivative of $F(z)$) and using (21) we finally obtain for $F(z)$ in the region $z \geq z_I$

$$\frac{\partial^2 F}{\partial z^2} = (z - z_I)\frac{3\nu_{tr}\nu'_{il}}{v_{ex}^2}eE(z_I)F \tag{22}$$

where we use $v_{ex}^2 = 2W_{ex}/m$.

There are no inelastic collisions in the region $z \leq z_I$ and (12) reads

$$\frac{\partial}{\partial z}\frac{v^3(z)}{\nu_{tr}(z)}\frac{\partial}{\partial z}F = 0. \tag{23}$$

Solving (22) and (23) with boundary conditions (15), (18) and continuity conditions for $F(z)$ and its derivative at $z = z_I$ and using expressions (13) we find the following expression for $\gamma(W_0, E)$ (see the appendix):

$$\gamma(W_0, E) \equiv \frac{j_e}{j_i} \\ = G(W_0)\gamma_i\left(1 + \frac{3}{4}\frac{W_0}{W_{ex}}\left(\frac{\sigma_{tr}(W_{ex})}{\sigma'_{il}(W_{ex})eE\lambda_{tr}}\right)^{1/3} \right. \\ \left. + \frac{3}{4v_0}\int_0^{z_I}\frac{\nu_{tr}(z)\,\mathrm{d}z}{(1 + eEz/W_0)^{3/2}}\right)^{-1}. \tag{24}$$

This expression generalizes the results of [7]. In this derivation we assumed that the distance between electrodes L is much larger than z_I. To obtain the result for the opposite case, $L < z_I$, we can simply consider the anode as a layer of completely absorbing particles ($\sigma'_{il}[W(L)] \to \infty$) placed at the distance L from the cathode. Then, we return to the result of [7]:

$$\gamma = G(W_0)\gamma_i\left(1 + \frac{3}{4v_0}\int_0^L \frac{\nu_{tr}(z)\,\mathrm{d}z}{(1 + eEz/W_0)^{3/2}}\right)^{-1}. \tag{25}$$

As in [7] from the integration over z in (24) we switch to the integration over the electron kinetic energy W, $W = W_0 + eEz$.

$$\gamma = G(W_0)\gamma_i\left(1 + \frac{3}{4}\frac{W_0}{W_{ex}}\left(\frac{\sigma_{tr}(W_{ex})}{\sigma'_{il}(W_{ex})eE\lambda_{tr}}\right)^{1/3} \right. \\ \left. + \frac{3N}{4eE}W_0\int_{W_0}^{W_{ex}}\frac{\sigma_{tr}(W)\,\mathrm{d}W}{W}\right)^{-1}. \tag{26}$$

One can rewrite the last expression in a more elegant form:

$$\gamma(W_0, X) = G(W_0)\gamma_i\frac{1}{1 + w\alpha(\tilde{X}/X)^{1/3} + \psi(w)\tilde{X}/X} \tag{27}$$

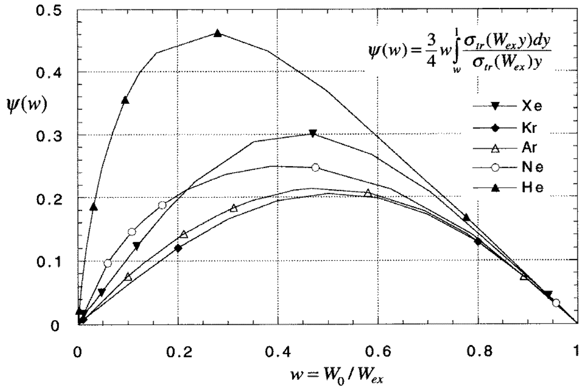

Figure 1. Functions $\psi(w)$ for all noble gases.

Table 1. Parameters determining differential effective secondary emission coefficient.

Gas	$\tilde{X} \equiv \tilde{E}/p$ (V Torr^{-1})	α	a	b	$\sigma_{il}(W)$ (10^{-18} cm^2) ($\sigma'_{il}(W_{ex})(W - W_{ex})$)
He	185	1.27	7.53	8.3	$2.7(W - 20)$
Ne	153	1.79	1.7	1.48	$1.25(W - 16)$
Ar	566	2.0	0.87	0.06	$9(W - 11.5)$
Kr	787	1.44	0.80	0.0	$25(W - 11.3)$
Xe	710	1.36	1.33	0.33	$33(W - 10)$

where $w = W_0/W_{ex}$, and X is the reduced magnitude of the electric field,

$$X \equiv E/p. \tag{28}$$

Here we denoted the characteristic electric field reduced to a pressure of 1 Torr as $\tilde{X}$:

$$\tilde{X} = \frac{W_{ex}}{e\tilde{\lambda}_{tr}(W_{ex})} \equiv (W_{ex}/e)\sigma_{tr}(W_{ex}) \times 3.53 \times 10^{16} \tag{29}$$

and the electron mean free path (m.f.p.) at 1 Torr ($N = 3.53 \times 10^{16}$ cm^{-3}) as $\tilde{\lambda}_{tr}(W_{ex})$. The function $\psi(w)$ and coefficient α are

$$\psi(w) = \frac{3}{4}w\int_w^1 \frac{\sigma_{tr}(W_{ex}y)\,\mathrm{d}y}{\sigma_{tr}(W_{ex})y}$$

$$\alpha = \frac{3}{4}\left(\frac{\sigma_{tr}(W_{ex})}{\sigma'_{il}(W_{ex})W_{ex}}\right)^{1/3}. \tag{30}$$

Plots of the function $\psi(w)$ for noble gases are given in figure 1. With a high accuracy it can be presented in the form

$$\psi(w) \approx a\frac{w(1 - w)}{1 + bw} \tag{31}$$

convenient for numerical simulations. The values of $\tilde{X}$, α, and coefficients a, b for all noble gases, together with approximations we use for $\sigma_{il}(W)$ near the thresholds, are given in table 1.

After integration of (27) over W_0, we find the resulting ESEC as a function of electric field.

$$\gamma(X) = \int_0^{W_{max}} \gamma(W_0, X)\,\mathrm{d}W_0. \tag{32}$$

Equations (26) (or (27)) and (32) represent the main result of this paper.

For the quantitative example of utilizing expressions (27) and (32) we consider two cases, which we name (i) the He-like case, when the distribution of emitted electrons is wide and most of them are emitted in the energy range where the function $\psi(w)$ also has a maximum and changes very little over this energy range [9], and (ii) the Xe-like case, when all emitted electrons are concentrated at low energies, so that the maximum of their energy, W_{max}, is less than the energy at which the function $\psi(w)$ has its maximum [9]. Each of these conditions can be met in any gas, but the first case is more typical for He and Ne, while the second case is usually met in Ar, Kr and Xe.

In the first case we substitute $\psi(w)$ by its average value $\bar{\psi}(w_{max}) = \int_0^{W_{max}/W_{ex}} \psi(w)G(W_{ex}w)W_{ex}\,\mathrm{d}w$; in the second case we substitute $\psi(w)$ by the linear function $\psi(w) = w\bar{\psi}'$, where $\bar{\psi}' \equiv \psi(w_{max})/w_{max}$ is the average value of the derivative in the interval 0–W_{max}.

In both cases we assume that the function $G(W_0)$ is constant for the energies below the maximum energy, W_{max}, and is zero above it. Using the normalization condition (16) for $G(W_0)$ we find that $G(W_0) = 1/W_{max}$, for $W_0 < W_{max}$.

(i) *He-like case.* $\psi(w) = \bar{\psi}(w_{max})$. Integrating (27) we obtain

$$\bar{\gamma}(X) = \gamma_i \frac{1}{\alpha w_{max}} \left(\frac{X}{\tilde{X}}\right)^{1/3} \ln\left[\left(1 + \bar{\psi}(w_{max})\frac{\tilde{X}}{X} + \alpha w_{max}\left(\frac{\tilde{X}}{X}\right)^{1/3}\right) \Big/ \left(1 + \bar{\psi}(w_{max})\frac{\tilde{X}}{X}\right)\right]. \tag{33}$$

For low electric fields one can neglect the unity compared to other terms in the logarithmic term and we have

$$\bar{\gamma}(X) = \frac{\gamma_i}{\alpha w_{max}}\left(\frac{X}{\tilde{X}}\right) \ln\left[1 + \frac{\alpha w_{max}}{\bar{\psi}}\left(\frac{X}{\tilde{X}}\right)^{2/3}\right] \approx \gamma_i \frac{1}{\bar{\psi}}\frac{X}{\tilde{X}}\left[1 - \frac{\alpha w_{max}}{2\bar{\psi}}\left(\frac{X}{\tilde{X}}\right)^{2/3}\right]. \tag{34}$$

(ii) *Xe-like case.* $\psi(w) = w\bar{\psi}'$. Integrating (27) from 0 to w_{max} we obtain

$$\bar{\gamma}(E/p) = \frac{\gamma_i}{\beta w_{max}} \ln(1 + \beta w_{max}) \tag{35}$$

where

$$\beta w_{max} \equiv \psi(w_{max})\frac{\tilde{X}}{X} + \alpha w_{max}\left(\frac{\tilde{X}}{X}\right)^{1/3}. \tag{36}$$

For low field one can neglect the unity compared to βw_{max} in the logarithm term and we have

$$\bar{\gamma}(E/p)/\gamma_i = (1/\beta w_{max}) \ln \beta w_{max} \approx \frac{(X/\tilde{X})}{\psi(w_{max})} \ln\frac{\psi(w_{max})}{(X/\tilde{X})}. \tag{37}$$

For high field, $\beta w_{max} \ll 1$, expression (35) gives $\bar{\gamma}(X)/\gamma_i = 1 - \beta w_{max}/2$.

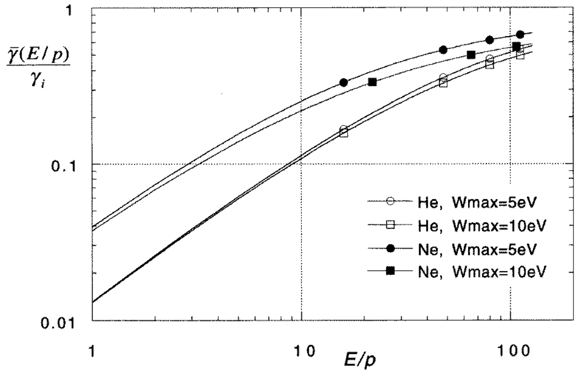

Figure 2. ESEC for 5 and 10 eV electrons ejected in He and Ne.

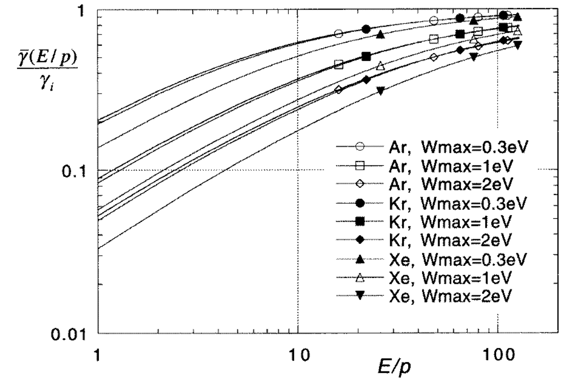

Figure 3. ESEC for 0.3–2 eV electrons ejected in Ar, Kr and Xe. The values of ESEC for Ar and Kr are virtually indistinguishable.

Although the derivation of the ESEC is not valid in the region of high electric field, the expressions (27), (33) and (35) have qualitatively correct asymptotic behaviour, so one can use them in all range of electric fields. The plots of $\bar{\gamma}(E/p)/\gamma_i$ for different gases and different cathode materials (any combination of gas–cathode material is characterized by the values of w_{max}) are presented in figures 2 and 3.

4. Effective secondary emission coefficient for a mixture of noble gases

In many applications the combination of a few noble gases, typically two or three, rather than a single pure gas is used as a working gas. In this case the ESEC defined as a ratio of electron and ion currents at the cathode depends not only on the electric field at the cathode and gas composition, but also on the dynamics of the specific discharge. Indeed, by definition

$$\bar{\gamma} = \frac{1}{j}\sum_k \gamma_k^{mix} j_k = \sum_k \gamma_k^{mix}\frac{j_k}{j} \tag{38}$$

where j_k is a partial ion current related to a 'k' gas component, and γ_k^{mix} is a partial ESEC for this current component in the mixture. The partial ESEC γ_k^{mix} depends only on the electric field E/p and gas composition, but the ratios j_k/j are determined by the discharge in the whole gap and may even vary with time when E/p stays constant. Thus, it is more practical in this case to use γ_k^{mix} instead of $\bar{\gamma}$.

In a general case of an arbitrary gas mixture the values of γ_k^{mix} can be found only in numerical kinetic simulations, such as direct Boltzmann or PIC-MC simulations. There are, however, some cases when an expression for γ_k^{mix} can be found analytically with relative ease. We consider here two cases, both for two-component mixtures, which are often used in practical applications. In the first case the component with the lowest excitation threshold has such a small partial pressure that one can neglect electron collisions with its atoms. The ionization mechanism for this component is mainly due to Penning ionization by excited atoms of main component. In the second case the partial pressure of the component with the lowest excitation threshold is large enough that the electron distribution function drops significantly in the region close to the excitation threshold of this component. This kind of mixture is often used in light emitting devices, like plasma displays or in new xenon based lamps.

We assume that in both cases the spectrum of secondary electrons related to the component with low excitation threshold is narrow and the spectrum of secondary electrons related to the component with high excitation threshold is wide. This means that one of γ_k^{mix} will behave as Xe-like, the other as He-like. For convenience, in the following description we will consider helium as a high-pressure component and xenon as an admixture in both cases. One can easily change notations He to Ne, and Xe to Ar or Kr in final expressions.

In the first case Xe admixture does not affect electron scattering, thus γ_{He}^{mix} will be the same as in pure helium, while γ_{Xe}^{mix} will be determined by a narrow spectrum of secondary electrons (related to the action of Xe ions) and by electron scattering on He atoms. This means that one can use expressions (35)–(37) for γ_{Xe}^{mix}, but use functions $\tilde{X}_{He}$, $\psi_{He}(w_{max})$ and α_{He} related to He and as w_{max} use the ratio of maximum energy of emitted electrons (due to the action of xenon ions), W_{max}^{Xe}, to the excitation energy of He atoms (W_{ex}^{He}): $w_{max} = W_{max}^{Xe}/W_{ex}^{He}$.

In the second case the xenon pressure is high enough so that beyond the xenon excitation threshold distribution function drops fast and one can neglect inelastic processes between electrons and helium atoms. The only process that helium atoms contribute in this case (even at high helium pressure) is elastic scattering of electrons. Thus, this mixture can be represented as a modified Xe gas density N_{Xe}, with atomic elastic cross-section $\sigma_{tr} = \sigma_{tr}^{Xe} + (N_{He}/N_{Xe})\sigma_{tr}^{He}$, and inelastic cross-section $\sigma_{il} = \sigma_{il}^{Xe}$. Sometimes it is more convenient to consider this gas as a surrogate gas density $N = N_{He} + N_{Xe}$, and macroscopic cross-sections $\Sigma_{tr} \equiv N\sigma_{tr} = N_{Xe}\sigma_{tr}^{Xe} + N_{He}\sigma_{tr}^{He}$ and $\Sigma_{il} \equiv N\sigma_{il} = N_{Xe}\sigma_{il}^{Xe}$.

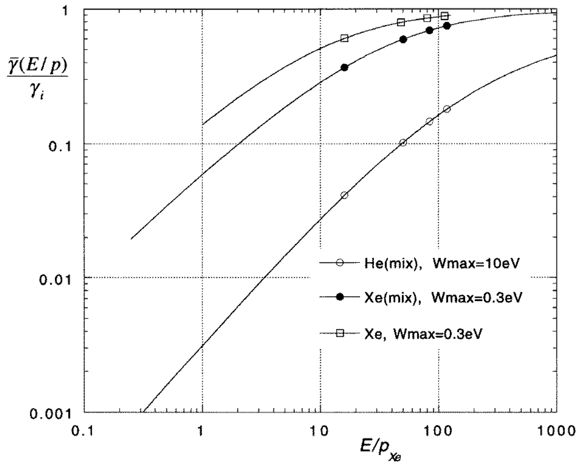

Figure 4. Partial ESECs in the gas mixture of 90% He + 10% Xe and in pure Xe.

Now one can repeat a consideration similar to the one made in the previous section and obtain an expression for differential ESEC similar to (27)

$$\gamma_{Xe,He}^{mix}(W_0, X_{mix}) = G_{Xe,He}(W_0)\gamma_i^{Xe,He} \times \left[1 + w\alpha_{mix}\left(\frac{\tilde{X}_{mix}}{X_{mix}}\right)^{1/3} + \psi_{mix}(w)\frac{\tilde{X}_{mix}}{X_{mix}}\right]^{-1} \tag{39}$$

where $G_{Xe,He}(W_0)\gamma_i^{Xe,He}$ are differential vacuum secondary emission coefficients for appropriate ions, $w = W_0/W_{Xe}$, $X_{mix} = E/p_{Xe}$,

$$\tilde{X}_{mix} = \tilde{X}_{Xe} + \frac{W_{Xe}}{W_{He}}\frac{N_{He}}{N_{Xe}}\tilde{X}_{He} \tag{40}$$

and α_{mix} and $\psi_{mix}(w)$ can be expressed in terms of individual functions as

$$\alpha_{mix} = \alpha_{Xe}\left(1 + \frac{\sigma_{tr}^{He}(W_{Xe})N_{He}}{\sigma_{tr}^{Xe}(W_{Xe})N_{Xe}}\right)^{1/3} \equiv \alpha_{Xe}\left(\frac{\Sigma_{tr}^{mix}(W_{Xe})}{\Sigma_{tr}^{Xe}(W_{Xe})}\right)^{1/3} \tag{41}$$

$$\psi_{mix}(w) = \frac{\Sigma_{tr}^{Xe}(W_{Xe})}{\Sigma_{tr}^{mix}(W_{Xe})}\psi_{Xe}(w) + \frac{\Sigma_{tr}^{He}(W_{He})}{\Sigma_{tr}^{mix}(W_{Xe})}\frac{W_{He}}{W_{Xe}} \times \left[\psi_{He}\left(w\frac{W_{Xe}}{W_{He}}\right) - w\psi_{He}\left(\frac{W_{Xe}}{W_{He}}\right)\right]. \tag{42}$$

Here we introduced individual macroscopic transport cross-sections for each component:

$$\Sigma_{tr}^{Xe,He} = N_{Xe,He}\sigma_{tr}^{Xe,He}.$$

After integrating (39) over the spectrum of secondary electrons for each gas component we obtain (33) and (34) for γ_{He}^{mix} and (35) and (36) for γ_{Xe}^{mix}. For the illustration in figure 4 we present plots of the γ_{Xe}^{mix} and γ_{He}^{mix} in the mixture of 90% He and 10% Xe described by the formulae (33)–(36) and (40)–(42) and the plot of the ESEC in pure Xe with the same energy spectra of emitted electrons.

5. Summary

We have investigated the influence of the electric field on the effective secondary electron emission coefficient in a high-density noble gas. This effect was first explained qualitatively by J J Thomson and L B Loeb as a result of joint action of the secondary emission of electrons from the surface, back-reflection of these electrons by the gas atoms and their extraction from the cathode vicinity by the electric field. They assumed that all of these processes were independent of the processes in the bulk of the discharge.

Later, the present authors [7] showed analytically that in most cases the whole concept of the ESEC (or the second Townsend coefficient) as a property of the surface–gas interaction, determined in the close vicinity of the surface and independent of the discharge in the bulk, should be reconsidered. For example, in some cases the processes in the cathode vicinity and in the bulk of the gas discharge are so tightly interwoven that the ESEC is determined by the discharge in the whole gap. Since electron dynamics plays such an important role in establishing the ESEC, one should expect that its value will be different in the stationary and non-stationary cases.

In this paper we have shown that if the voltage applied to a gap is significantly higher than the excitation/ionization potential of the gas atoms, then the size of the region in which processes affect ESEC is much smaller than the gap length (but much larger than electron mean free path).

In order to obtain some quantitative results, we limited our consideration to the case of a high-density noble gas and stationary, uniform and weak electric field ($eE\lambda_e \ll W_0$). The general case of arbitrary field strength requires numerical consideration. A specifics of noble gases, which was essential to our consideration, is that the width of the energy spectrum (maximum energy) of secondary electrons emitted from the surface is less than the excitation threshold of the gas atoms. Then, when an electron experiences inelastic collision its energy changes so much that it immediately leaves the initial energy range and is not able to return to the cathode surface. In contrast, in many molecular gases the excitation threshold is determined by the molecular rotational and vibrational terms and is often much less than the width of the energy spectrum of the secondary electrons, which is still determined by the ionization potential of an atom or a molecule.

The expressions (27) and (31) for the differential and total ESEC (and (32) or (34)), are applicable not only for the case when the electron emission is caused by the action of ions, but also for any other source (photons, metastables, etc), which can be characteristized by the yield δ_e (instead of γ_i) and the spectrum $G_e(W_0)$ (instead of $G(W_0)$) of the secondaries. Then $\gamma(E/p, E_{max})/\gamma_i$ and $\delta(E/p, W_{max})/\delta_e$ are the same if the spectra $G(W_0)$ and $G_e(W_0)$ are identical.

For comparison we plot in figure 5 experimental data [4–6, 11] of the ESEC for the low-energy (below 1.5 eV) photo-electrons emitted in argon and the result obtained with (34). At very low E/p our theory differs significantly form the experimental data [4], probably because at such low E/p one cannot neglect the energy losses in elastic collisions. At higher E/p our theory is in good agreement with the fit of experimental data of Felsch and Pech [5, 6], which they suggested:

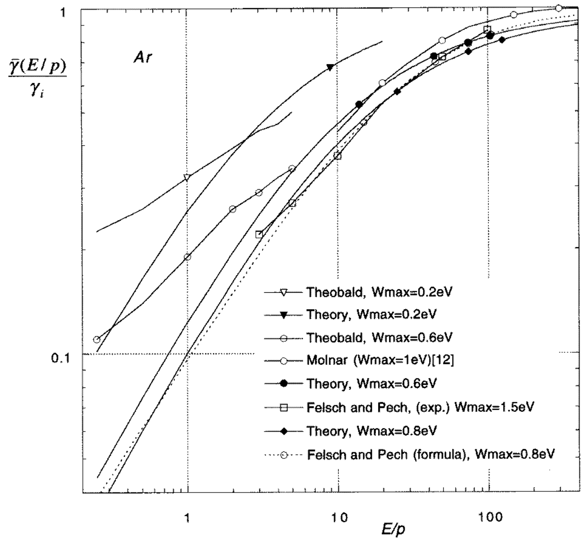

Figure 5. ESEC in Ar. Comparison with experiment. The dotted line is the plot of (43) with $p\lambda_e = 1/37$ and $W_{max} = 1.5$ eV, as well as with $p\lambda_e 1/49$ and $W_{max,eff} = 0.8$ eV.

$$\gamma_{FP}(E/p)/\gamma_i = (1 + 0.0948/\eta')^{-2/3} \tag{43}$$

where $\eta' = K(e/W_{max})^{2/3}(p\lambda_e)^{3/2}(E/p)$, and $K = 1\ V^{-1/3}$ (Torr cm)$^{-1/2}$. However, in order to use in this expression the *maximum energy of photoelectrons* they used a value of the parameter $p\lambda_e$ higher than we do: 1/37 for Ar instead of 1/49 ($p\lambda_{tr}(W_{ex}) = W_{ex}/\tilde{X} = 1/49$). We believe that this can be explained by the peculiarities of the energy spectrum of the photoelectrons, which has a relatively long tail, which does not contribute to the ESEC, so that *effective maximum energy* is closer to the average energy of the photoelectrons rather than to their maximum energy. Comparison with data of Molnar [11] supports the same conclusions: our formula gives good agreement with experiment if we use the average energy of the photoelectrons instead of their maximum energy for the value of the W_{max} in our *model spectrum* of secondary electrons (see figure 5). As we noted at the end of section 3, at high E/p (such that $eE\lambda_{tr} > W_{max}$) our theory is not valid, but, as one can see from figure 5, the error it gives does not exceed 10%.

As we mentioned in introduction we assumed that the electric field changes slowly enough and its gradient is small. Using the found solution we can formulate these conditions quantitatively. As one can see from the solution for the function $F(z)$ (see (A1)), the size of the region which determines the ESEC can be estimated as

$$\ell_\gamma \sim z_{I.max} + \kappa^{-1} \approx \frac{W_{ex}}{eE} + \left(\frac{v_{ex}^2}{3\nu_{mt}\nu'_{il}(W_{ex})eE}\right)^{1/3}.$$

It takes a time of about

$$\tau_\gamma \sim \ell_\gamma / v_d$$

for an electron to drift across this region. One may consider that the electric field changes slowly in time and space if

$$\tau_\gamma |\partial E/\partial t| \ll E \qquad \ell_\gamma |\nabla E| \ll E.$$

Acknowledgments

The authors are grateful to Professor Art Phelps for his interest in this work and for sharing with us his compilation of experimental data [4–6, 11] of the ESEC and to Professor D D Ryutov for discussions and useful comments.

Appendix

Let us denote $\kappa = [(3\nu_{tr}\nu'_{il}/v_{ex}^2)eE(z_I)]^{1/3}$, then equation (22) for $z \geq z_I$ takes the form

$$\frac{\partial^2}{\partial \xi^2} F = \kappa^3 \xi F \tag{A1}$$

where $\xi = z - z_I$. This is an Airy equation, and its solution, which has an asymptote that goes to zero ($F \propto (\kappa\xi)^{-1/4} \exp[-(2/3)(\kappa\xi)^{3/2}]$) is

$$F(z \geq z_I) = a\,\mathrm{Ai}(\kappa\xi) \equiv \frac{a}{\pi}\sqrt{\kappa\xi/3}K_{1/3}(\tfrac{2}{3}(\kappa\xi)^{3/2}) \tag{A2}$$

where $\mathrm{Ai}(x)$ is the Airy function and $K_{1/3}$ is the modified Bessel function of the order 1/3.

In the region $z \leq z_I$ (or $\xi < 0$) using (23) we obtain for $F(z)$

$$F(z \leq z_I) = F(z_I)\left(1 + C\int_z^{z_I} \frac{\nu_{tr}}{v^3}\,\mathrm{d}z\right). \tag{A3}$$

At the point $z = z_I$ both function $F(z)$ and its first derivative must be continuous functions of z. Applying this conditions to (A2) and (A3) we obtain unknown constants a and C:

$$a = F(z_I)/\mathrm{Ai}(0) \approx 2.8F(z_I) \tag{A4}$$

$$C = \frac{\mathrm{Ai}'(0)}{\mathrm{Ai}(0)}\frac{\kappa v_{ex}^3}{\nu_{tr}(v_{ex})} = 0.73\frac{\kappa v_{ex}^3}{\nu_{tr}(v_{ex})} \tag{A5}$$

where Ai(0) and Ai′(0) are the values of the Airy function and its derivative at the zeroth argument. Using (13) and the condition that the electron current is constant between the cathode ($z = 0$) and the point $z = z_I$, we obtain

$$j_e(z \leq z_I) = \frac{2\pi e}{3}\frac{v^3}{\nu_{tr}}\frac{\partial F}{\partial z} = -\frac{2\pi e}{3}CF(z_I)$$

$$n_c = 2\pi v F(z_I)\left(1 + C\int_0^{z_I}\frac{\nu_{tr}}{v^3}\,\mathrm{d}z\right). \tag{A6}$$

Substituting these expressions into the boundary condition (18) and using the first of equations (13) we can write

$$\begin{aligned} j_e = -\frac{2\pi e}{3}CF(z_I) = G(W_0)\gamma_i j_i \\ +2\pi e\frac{v_0^2}{4}F(z_I)\left(1 + C\int_0^{z_I}\frac{\nu_{tr}}{v^3}\,\mathrm{d}z\right). \end{aligned} \tag{A7}$$

Solving the last of this series of equations with respect to $F(z_I)$ we find j_e:

$$j_e = j_i G(W_0)\gamma_i\left(1 + \frac{3}{4}\frac{v_0^2}{C} + \frac{3}{4}v_0^2\int_0^{z_I}\frac{\nu_{tr}}{v^3}\,\mathrm{d}z\right)^{-1}. \tag{A8}$$

Comparing this equation with the definition of γ, $\gamma = j_e/j_i$, we find the expression for $\gamma(W_0, E/p)$:

$$\begin{aligned} \gamma(W_0, E/p) &= G(W_0)\gamma_i\left(1 + \frac{3}{4}\frac{v_0^2}{C} + \frac{3}{4}v_0^2\int_0^{z_I}\frac{\nu_{tr}}{v^3}\,\mathrm{d}z\right)^{-1} \\ &= G(W_0)\gamma_i\Bigg[1 + \frac{3}{4}\frac{v_0^2}{v_{ex}^2}\left(\frac{\sigma_{tr}}{\sigma'_{il}eE\lambda_{tr}}\right)^{1/3} \\ &\quad + \frac{3}{4}v_0^2\int_0^{z_I}\frac{\nu_{tr}}{v^3}\,\mathrm{d}z\Bigg]^{-1}. \end{aligned} \tag{A9}$$

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